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APPLICATION OF 3D PRINTED MODELS FOR COUNTING PRISMS IN SOLIDS

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Abstract: *A system of tasks for counting prisms and the printed 3D models of solids from the tasks are presented. An example for construction of the 3d models is given. Ways of counting and recording the corresponding result of the tasks, as well as typical mistakes made by the solvers are considered. Results of an experiment with students in 5th and 6th grade from an elementary school are described. The presented system of tasks is suitable for use in classes in Mathematics, Information technology, Computer modeling, for work in STEM centers, etc. The auxiliary 3D models can be used when working with people with visual impairments.*
Keywords: *3D printing, mathematics education, didactic resources, manipulatives, digital competence, people with visual impairments*

1. INTRODUCTION

Tasks for counting squares and triangles in figures from classical in entertaining mathematics are becoming traditional in textbooks and math aids in Bulgarian primary school [9]. Such tasks are also offered to the National External Evaluation [4]. The reason is that solving them contributes to the development of spatial imagination, to the development of criticality and flexibility of thinking, to the formation of the ability to carry out research. When studying geometric bodies, it is also appropriate to solve tasks for counting them.
Educational aids continue to expand with the advent of new technologies in the additive technologies [6], [12]. The use of 3D printed manipulatives is particularly useful [3], [7], 8]. Through them, accessibility to relevant resources is ensured for people with visual impairments [2].

2. METHODS

Here will be presented a system of prism counting tasks, printed 3D models of the solids of the tasks, ways of counting and recording the result, expected mistakes made by the solvers and results of a conducted experiment.
In the month of May 2024, an experiment was conducted with students in 5th and 6th grade from 163 "Chernorizets Hrabar" elementary school, in Sofia city. Participants had the models from the task system on paper to record their answers.

3. EXPERIMENTAL

Task 1. Count the rectangular parallelepipeds in the solid (Table 1).

Table 1. 3D models of counting Tasks 1.1, 1.2 and 1.3

 1.1	 1.2	 1.3
https://3dplus.tonediko.com/wp-content/uploads/2024/05/Nsq.p1.stl	https://3dplus.tonediko.com/wp-content/uploads/2024/05/Nsq.p3.stl	https://3dplus.tonediko.com/wp-content/uploads/2024/05/Nsq.p2.stl

We recommend that solvers are left to count and indicate an answer by themselves. In case of an incorrect answer, a model of a rectangular parallelepiped can be provided and the description of the solution discussed. It is expected that there will be more than one way of counting. The goal is for the solvers to come to the idea of rationalizing the counting based on a given feature. One way to solve this is to sequentially count the rectangular parallelepipeds with bases - the horizontal squares. Figure 1 shows one such solution:

- 3 rectangular parallelepipeds with base "a" and heights 1, 2 and 3;
- 2 rectangular parallelepipeds with base "b" and heights 1 and 2;
- 1 rectangular parallelepiped with base “c” and height 1.

The number of rectangular parallelepipeds in the considered model is 3+2+1=6.

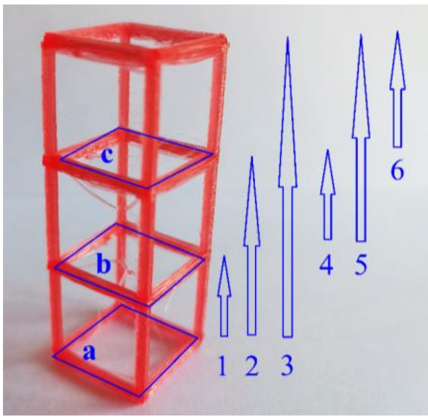


Figure 1. The rectangular parallelepipeds in the printed 3D model from Task 1.1

Another option for rational counting is to separate the rectangular parallelepipeds by species. For example, in the model from Task 1.2, there are 4 types of rectangular parallelepipeds, again considering a horizontal square as a basis (Fig. 2):

- 4 rectangular parallelepipeds with height 1;
- 3 rectangular parallelepipeds with height 2;
- 2 rectangular parallelepipeds with a height of 3;
- 1 rectangular parallelepiped with height 4,

or numerically expressed 4+3+2+1=10.

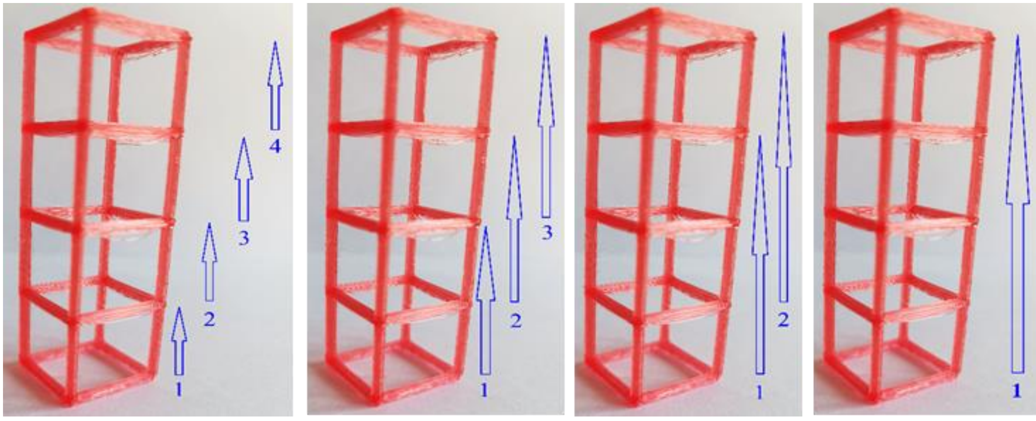


Figure 2. Solution of Task 1.2 according to the type of prisms

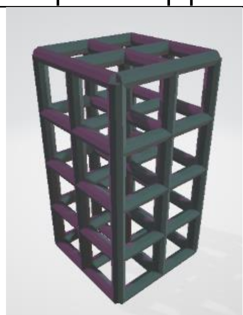
The following record may also be used:

- 4 rectangular parallelepipeds of the type 1x1x1;
- 3 rectangular parallelepipeds of the type 1x1x2;
- 2 rectangular parallelepipeds of the type 1x1x3;
- 1 rectangular parallelepiped of the type 1x1x4.

The number of rectangular parallelepipeds in the figure from Task 1.3 is equal to those from Task 1.2. The specific thing here is that the distances between the individual sections in the rectangular parallelepipeds are of different lengths. The expectation is that the solvers will fit the same number of intermediate sections in both models and use the solution of the previous task.

Task 2. Count rectangular parallelepipeds in the body (Table 2).

Table 2. 3D models of Tasks 2.1 and 2.2 for counting rectangular parallelepipeds

2.1 	2.2 
https://3dplus.tonediko.com/wp-content/uploads/2024/05/Nsq.p4.stl	https://3dplus.tonediko.com/wp-content/uploads/2024/05/Nsq.p5.stl

To solve Task 2.1, the already known number of rectangular parallelepipeds from Task 1.2 can be used. An incomplete answer of 20 is often arrived at, when considering the new composition as composed of 2 pasted models, as in Task 1.2. Such a build is easy to perform using the "Clone" menu button in the XYZmaker3DKit processing software (Fig. 3).

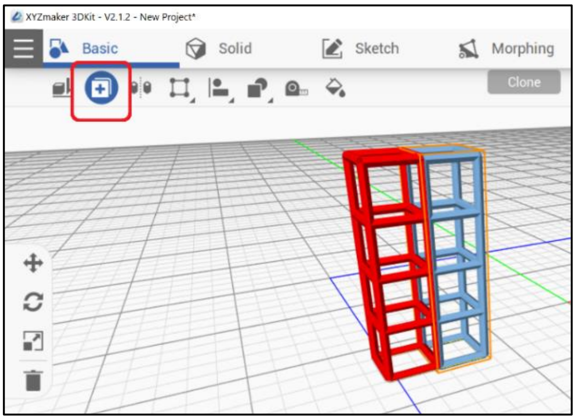


Figure 3. Creating the model from Task 2.1 by cloning the model from Task 1.2 in XYZmaker3DKit

In this case, the answer to Task 2.1 is 10x3=30, since 10 rectangular parallelepipeds with a 1x2 base must also be added.
By analogy, the model from Task 2.2 can be constructed. To create it, you can combine 4 solids from Task 1.2, or 2 solids from Task 2.1 (Fig. 4).

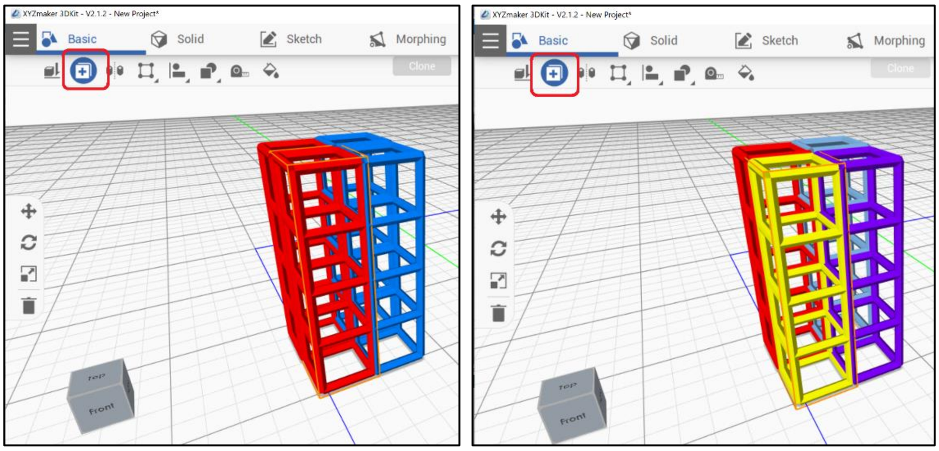


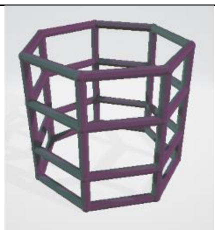
Figure 4. Creating the model from Task 2.2 using the model from Task 1.2 (left) and the model from Task 2.1 (right) in XYZmaker3DKit

A rational count using the model solution from Task 1.2 could be as follows:

- 40 rectangular parallelepipeds with a 1x1 base, when separating four rectangular parallelepipeds of the type 1x1x4, already considered in Task 1.2;
- 40 rectangular parallelepipeds with a 1x2 base, when separating four rectangular parallelepipeds of the type 1x2x4, already discussed in Task 2.1;
- 10 rectangular parallelepipeds with a 2x2 base.

Task 3. Count the prisms in the solid (Table 3).

Table 3. 3D figure models from Task 3 for counting prism

3.1 	3.2 	3.3 
https://3dplus.tonediko.com/wp-content/uploads/2024/05/Nhep.p1.stl	https://3dplus.tonediko.com/wp-content/uploads/2024/05/Nhep.p5.stl	https://3dplus.tonediko.com/wp-content/uploads/2024/05/Ntri.p5.stl
3.4 	3.5 	3.6 
https://3dplus.tonediko.com/wp-content/uploads/2024/05/Nhep.p2.stl	https://3dplus.tonediko.com/wp-content/uploads/2024/05/Nhep.p4.stl	https://3dplus.tonediko.com/wp-content/uploads/2024/05/Nhep.p3.stl

The "bases" of the solid given in the problem are different, but for counting the prisms in each of them, the horizontal "sections" are important. For example, solvers are expected to notice that the models in Task 3.3 and 3.4 contain the same number of prisms, even though their "base" is different. Some of the solvers are expected to realize that this number is known from the solution of Task 1.2 and make a generalization.
A DaVinci mini printer was used to print the models from the presented tasks. There was a typical drawback when printing three-dimensional figures - the loss of layers of the solis [1], (Fig. 5).

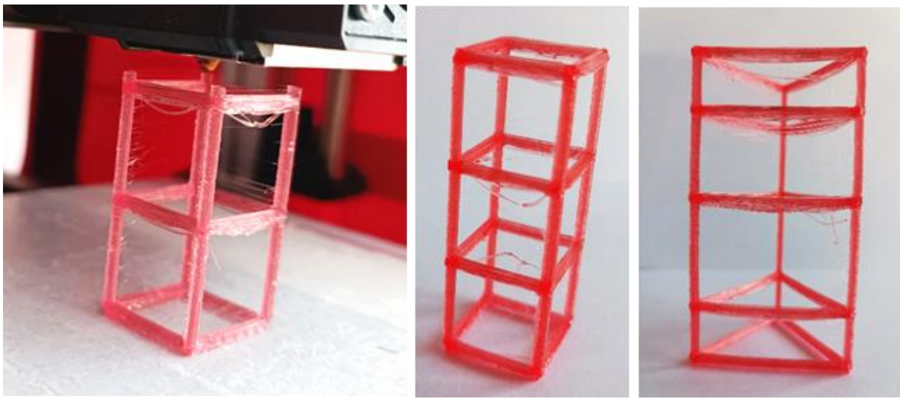


Figure 5. Fallen layers, when 3D printing three-dimensional models

When modeling the composition, it is good to take into account the thickness of the layer, depending on the material used, the type of printing and the intended goals [5], [10], [11], [13]. In this case, PLA was used, the lightness of which allows easy transportation and storage of the finished models, as well as work with them, including by people with visual impairments.

3.1. An experiment with students

During the experiment in 163 "Chernorizets Hrabar" School, Sofia city, 5th and 6th grade students had the tasks from the system on paper, on which they could work and record their answers.
The students used different options for noting intermediate results on paper by consecutively numbering the prisms found, enclosing already counted prisms, systematizing them by species (Fig. 6).

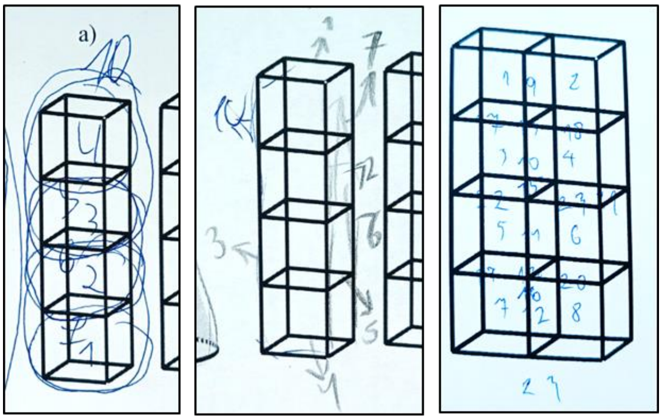


Figure 6. Variants of recording intermediate results of students

Correct answer 10 to Task 1.2 was given by 52% of the solvers, although some of them made an intermediate correction of the final number. It is noted that the most frequently given wrong answer is 9 (by 17% of solvers), which is usually the result of missing the "largest" parallelepiped. The next wrong answers are 4 and 5, which is the result of counting only the rectangular parallelepipeds 1x1x1 or also the rectangular parallelepiped 1x1x4. Several people indicated answer 7, there is also answer 12. The latter was probably obtained by poor organization of the census and double counting of some of the sites.
Only two students gave a correct answer to Task 2.1, with the predominant answer being numbers between 22 and 28. As expected, the answer 8 also occurs – the number of rectangular parallelepipeds of the type 1x1x1. For Task 2.2. the numbers 20, 40, 50, 60 are noticed as a solution, as well as 45. The students tried to count rationally using the solutions of previous problems. Half of the students answered Task 3.2 correctly. The number of prisms from Task 1.2 is the same as the number of prisms in Task 3.2 – a total of 10, and the students noticed this massively - those who indicated 9 as the answer to Task 1.2 also indicated 9 as the answer to Task 3.2.
We note that the 5th grade solvers had not yet studied a prism, only a cube and a rectangular parallelepiped. At the beginning of the lesson, the description of the prism solid was made, some examples were presented, the rectangular parallelepiped was evaluated as a type of prism, after which the students began independent work.
After they finished their work, a discussion was held about the solutions to the tasks and the counting methods. During discussion of the solutions, 3d printed models were provided, with which students demonstrated how they approached the counting and recorded the corresponding result.

4. CONCLUSION

The use of additive technologies in the study of solids helps solvers understand specific geometric constructions and mathematical concepts - both when working with ready-made 3D printed models and when creating them with a given software. Various 3D modeling, AR, VR software can be used to visualize the 3D models. The use of several types of technology helps to develop students' problem-solving skills. The presented system of tasks is suitable for use in classes in Mathematics, Information technology, Computer modeling, STEM centers and others. The 3D models are suitable for use when working with people with visual impairments.

5. ACKNOWLEDGMENTS

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